ECS455 Chapter 2

Cellular Systems

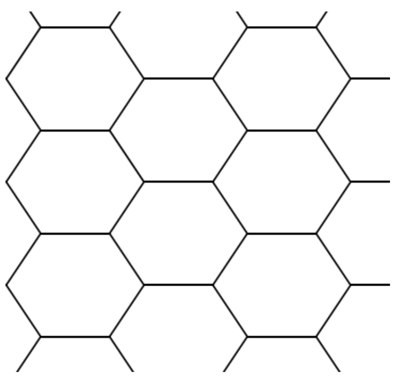
2.3 Sectoring

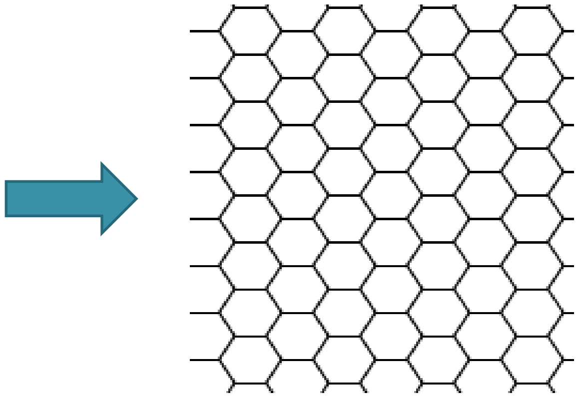
Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

Improving Coverage and Capacity

- As the demand for wireless service increases, the number of channels assigned to a cell eventually becomes insufficient to support the required number of users.
- At this point, cellular design techniques are needed to provide more channels per unit coverage area.
- Easy!?

$$C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$

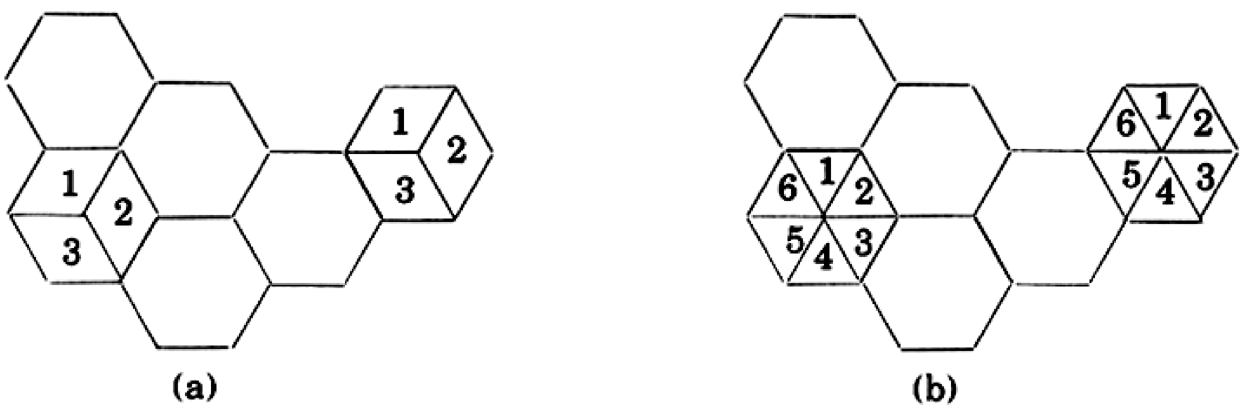




If cells can be reduced in size, more of them can be added in a given area, increasing the overall capacity.

Sectoring (N = 7)

- Ex. With no sectoring, suppose $m = 18$ channels/cell
  - With 120° sectoring, we have 6 channels/sector
  - With 60° sectoring, we have 3 channels/sector
- "Can support the same number of users" per cell
  - In the next section, we will consider different kind of capacity. For such capacity, sectoring will give less capacity.



[Rappaport, 2002]

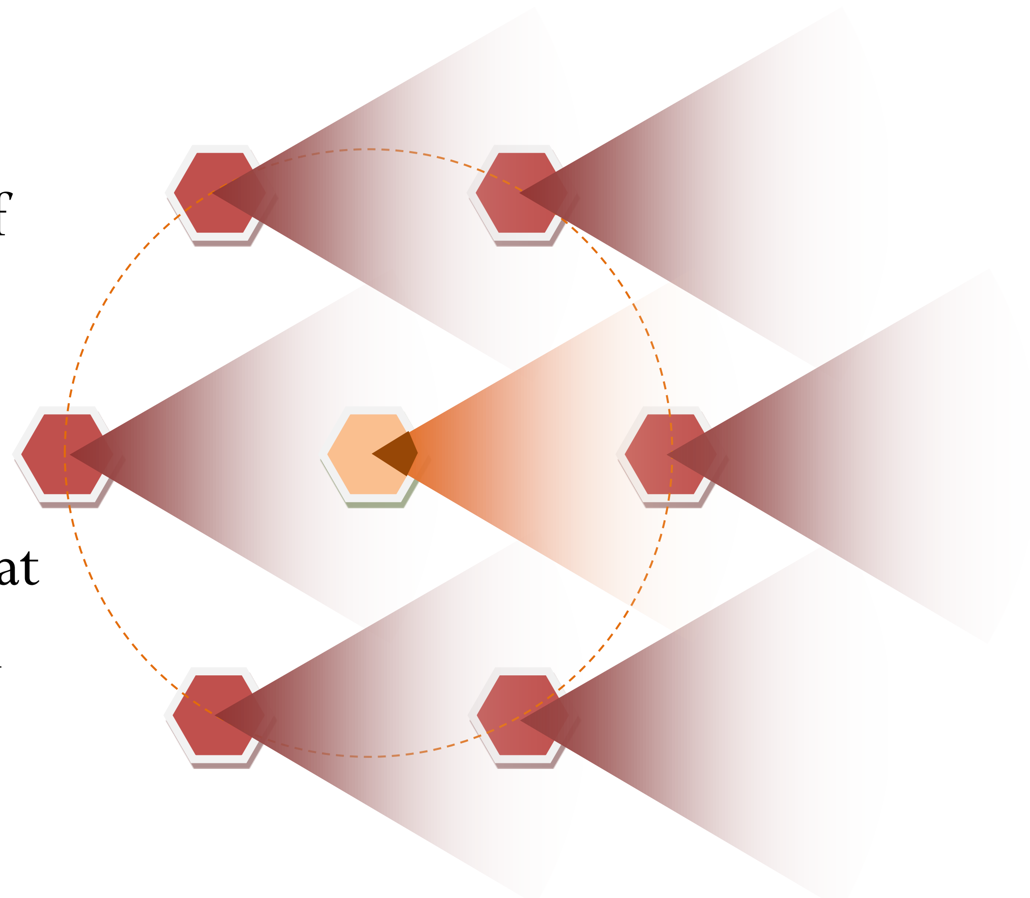
Figure 3.10 (a) 120° sectoring; (b) 60° sectoring.

- Why is this better?

60 Degree Sectoring

$$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

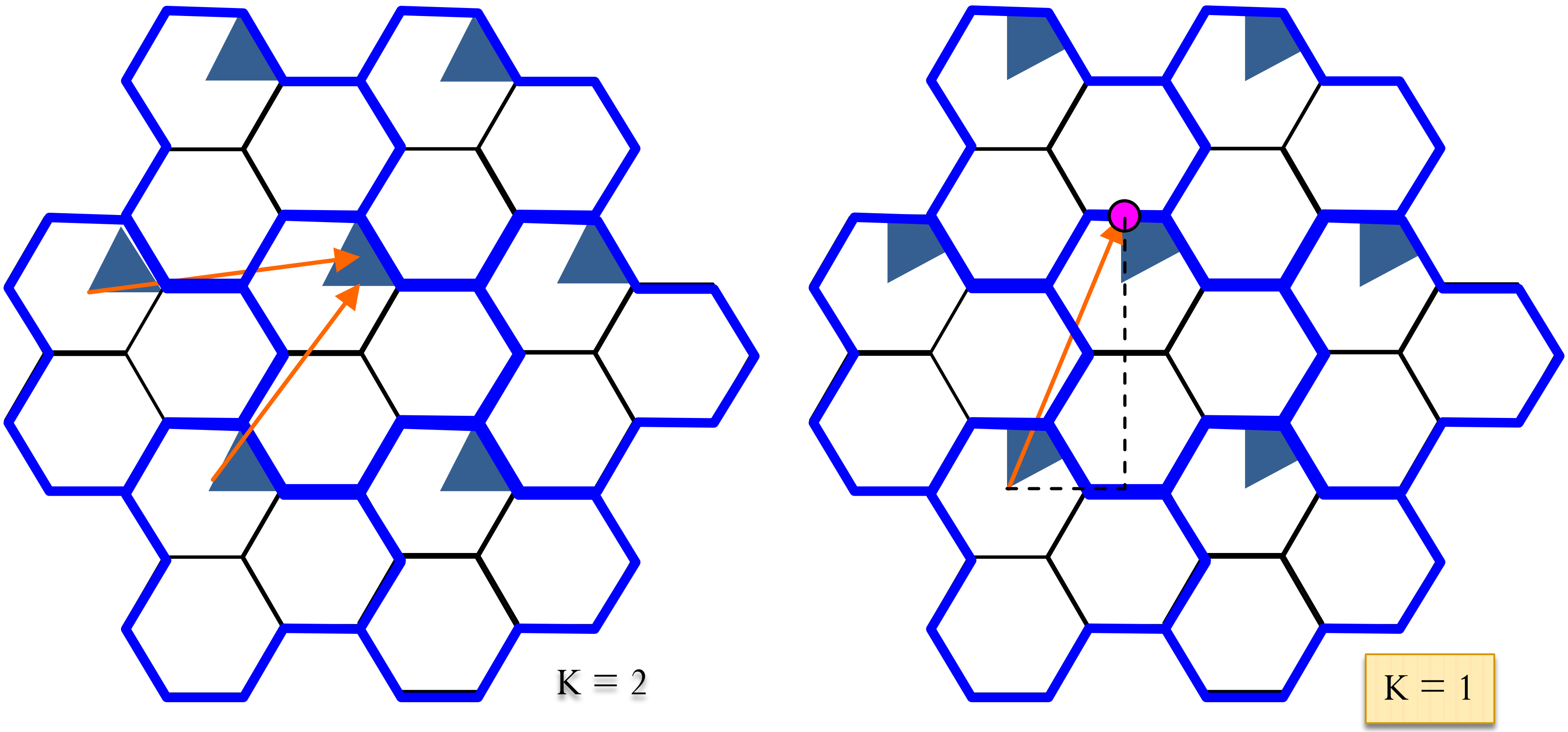
- Out of the 6 co-channel cells in the first tier, only one of them interfere with the center cell.
- If omnidirectional antennas were used at each base station, all 6 co-channel cells would interfere the the center cell.



The value of K changes from 6 to 1!

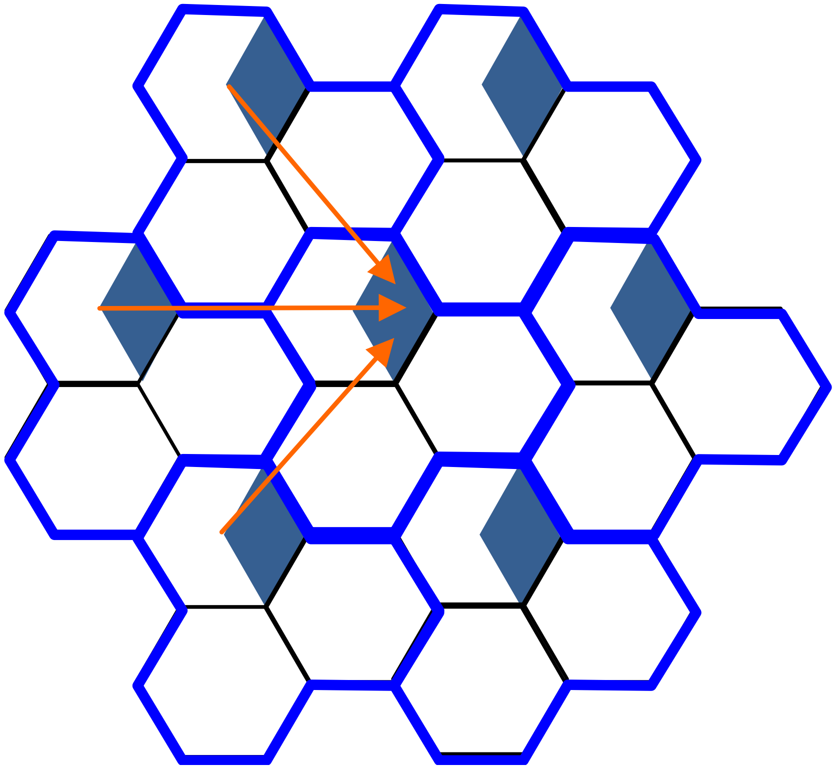
Sectoring (N = 3 , 60°)

$$SIR \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

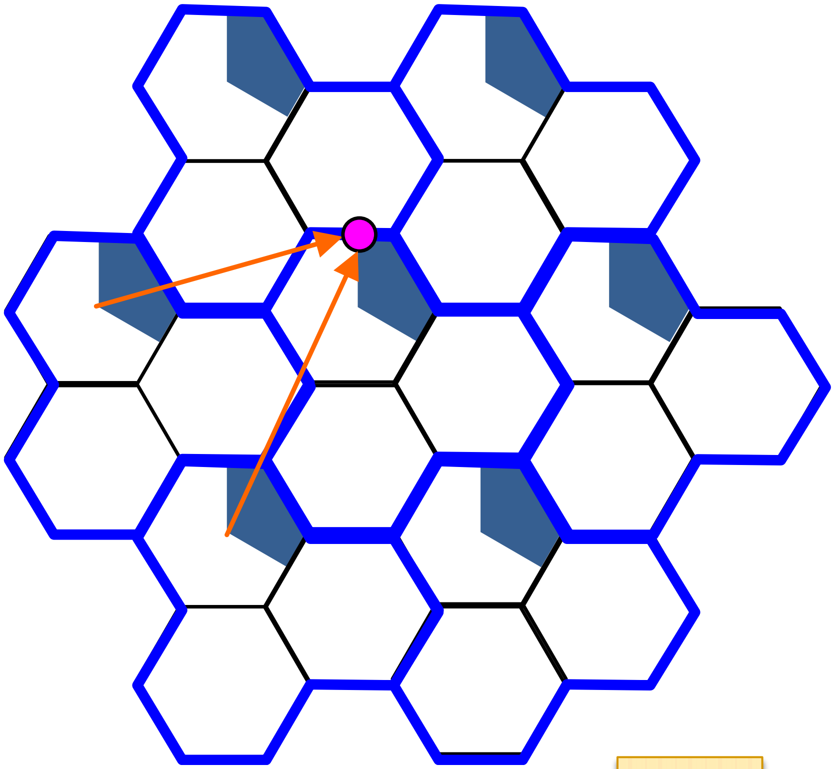


Sectoring (N = 3, 120°)

$$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$



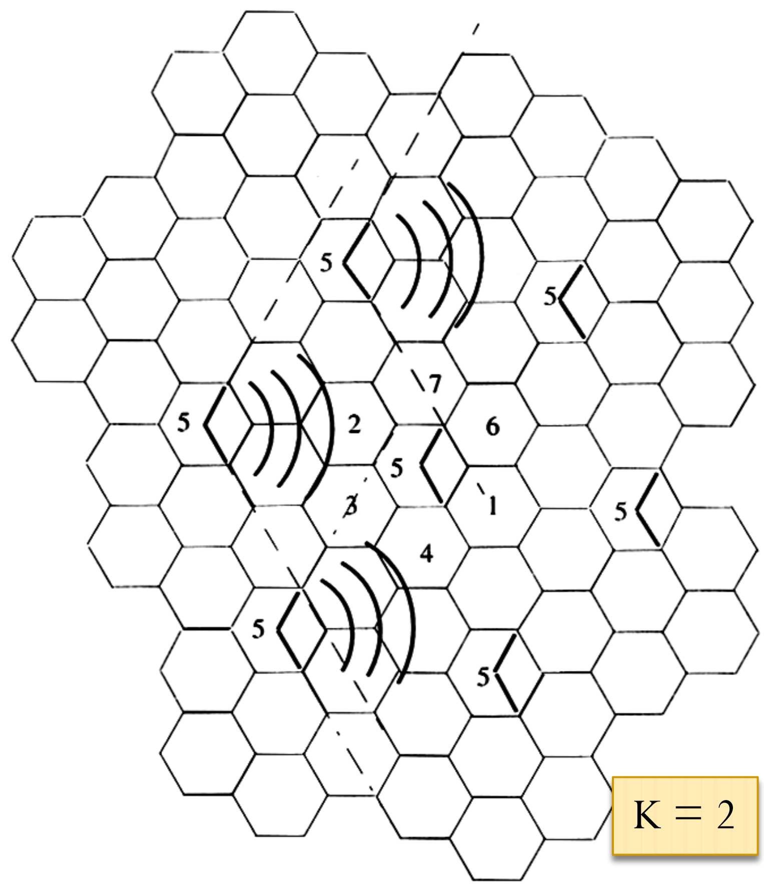
K = 3



K = 2

Sectoring (N = 7, 120°)

Assuming seven-cell reuse, for the case of 120° sectors, the number of interferers in the first tier is reduced from six to two.



[Rappaport, 2002, Fig 3.11]

$$\text{SIR} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma} \qquad C = \frac{A_{\text{total}}}{A_{\text{cell}}} \times \frac{S}{N}$$

Sectoring

- Advantages
  - Reduce interference by reducing K
    - Increase SIR (better call quality).
    - The increase in SIR can be **traded** with reducing the cluster size (N) which increase the capacity.

- Disadvantages
  - Increase number of antennas at each base station.
  - Next section: Decrease **trunking efficiency** due to channel sectoring at the base station.
    - The available channels in the cell must be subdivided and dedicated to a specific antenna.

ECS455 Chapter 2

Cellular Systems

2.4 Traffice Handling Capacity and Erlang B Formula

Dr.Prapun Suksompong
prapun.com/ecs455

Office Hours:
BKD 3601-7
Wednesday 15:30-16:30
Friday 9:30-10:30

Capacity Concept: A Revisit

- Q: If I have *m* channels per cell, is it true that my cell can support only *m* user?
- A: Yes and No
- Let's try one example.
- How often do you make a call?
  - 3 calls a day, on average.
- How long is the call?
  - 10 mins (per call), on average.
- So, one person uses

Capacity Concept: A Revisit

- If we can "give" the time that "User 1" is idle to other users,
  - then one channel can support users!!
- True?

New Concepts

- Using *m* as the capacity of a cell is too small.
- We can let more than one user share a channel by using it at different times.
- The number of users that a cell can support can then exceed *m*.
- Call initiation times are random
- **Blocked calls**
- Probability of (call) blocking P_b
  - the likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%

Trunking

- Allow a large number (*n*) of users to **share** the relatively small number of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.
- Exploit the **statistical behavior** of users.
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**.
  - Dimensionless
  - Denoted by A.
- **Holding Time**: Average duration of a typical call.
  - Denoted by $\mathbf{H = 1/\mu}$.
- **Request Rate**: The average number of call requests per unit time. Denoted by λ.
- Use A_u and λ_u to denote the corresponding quantities for one user.
- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where n is the number of users supported by the pool (trunked channels) under consideration.

Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion. Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).
  - The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

M/M/m/m Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channels are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.
- **The duration of the time that a user occupies a channel is *exponentially distributed***, so that longer calls are less likely to occur.
- There are m channels available in the trunking pool.
  - For us, m = the number of channels for a cell (C) or for a sector

Erlang B Formula

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

m = Number of trunked channels

Call blocking probability

$A =$ traffic intensity or load [Erlangs]

$= \frac{\lambda}{\mu}$

λ = Average # call attempts/requests per unit time

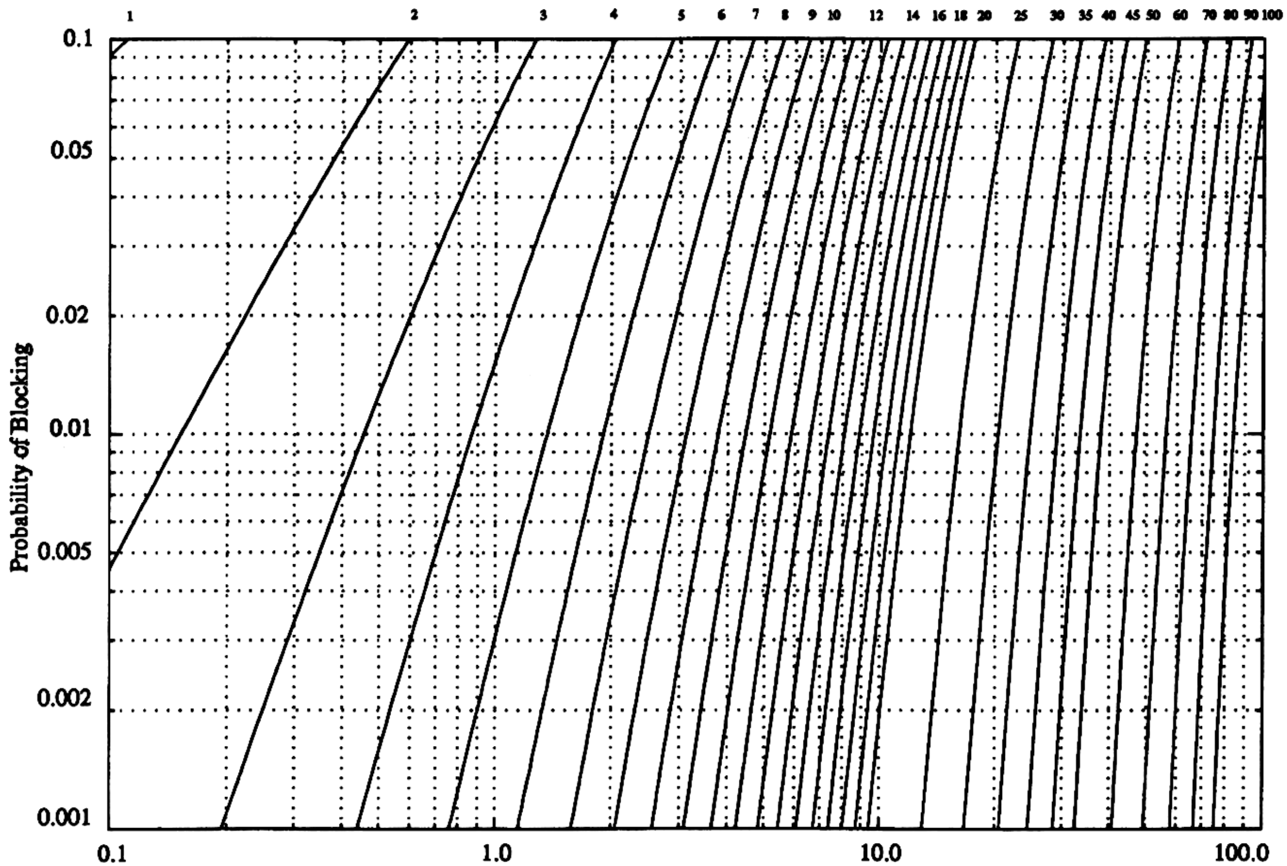
$\frac{1}{\mu} = H =$ Average call length

In MATLAB, use `erlangb(m,A)`

Erlang B Formula and Chart

$$P_b = \frac{\dfrac{A^m}{m!}}{\displaystyle\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Number of Trunked Channel (m)



Traffice Intensity in Erlangs (A)

(log-log plot)

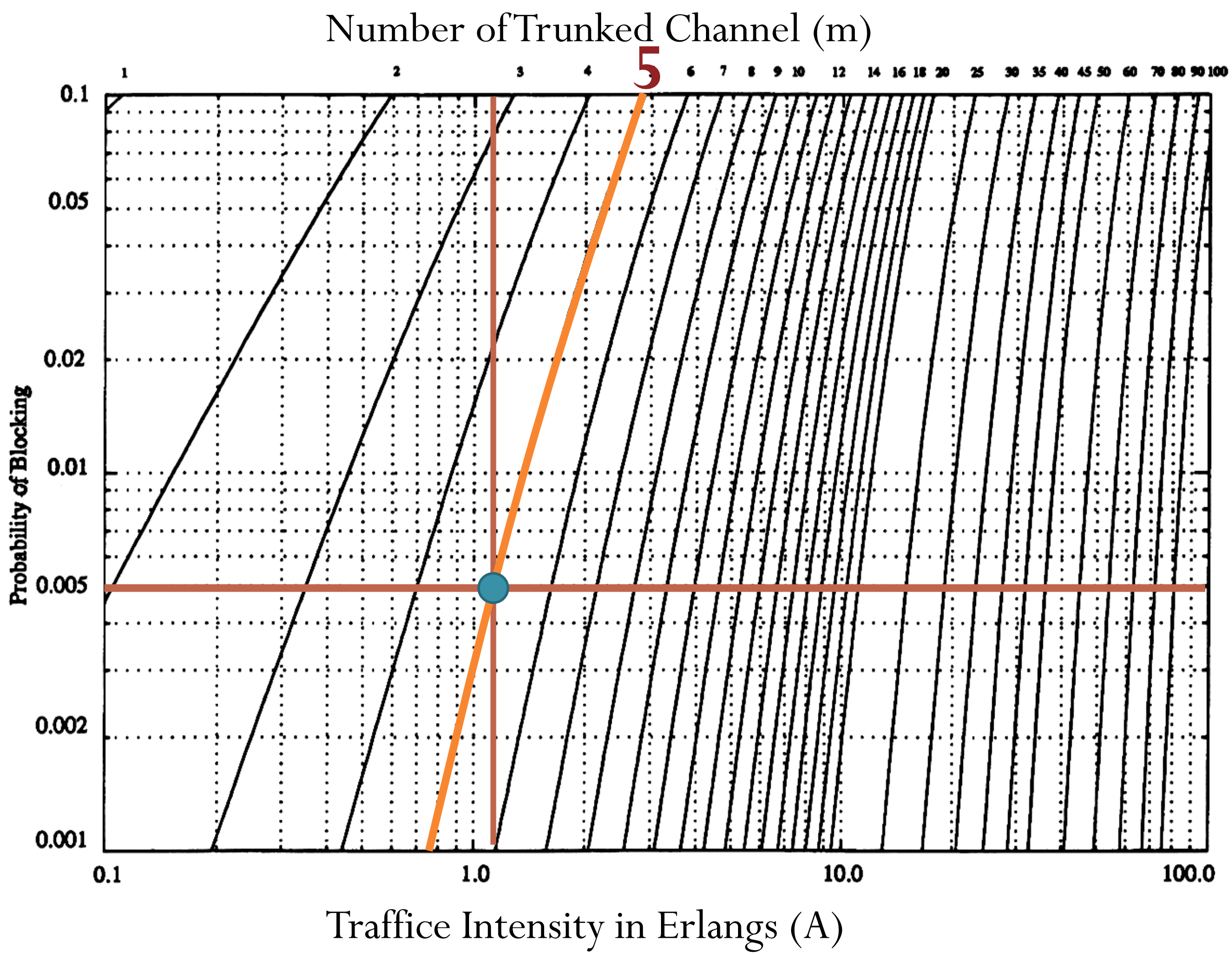
Example 1

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

  (a) 5

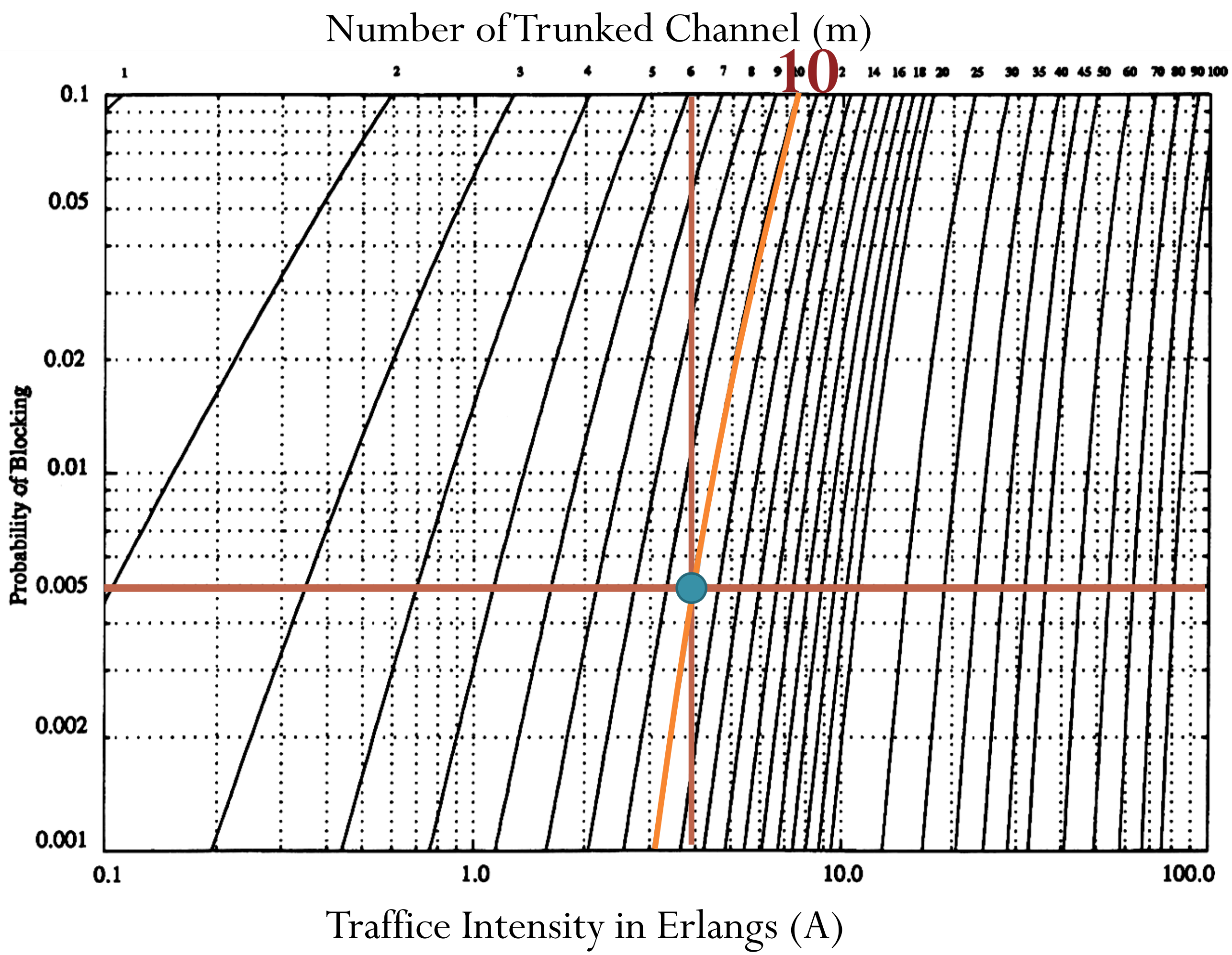
  (b) 10
- Assume each user generates $A_u = 0.1$ Erlangs of traffic.

Example 1a



$A \approx 1 \Rightarrow n \approx 10$ users

Example 1b



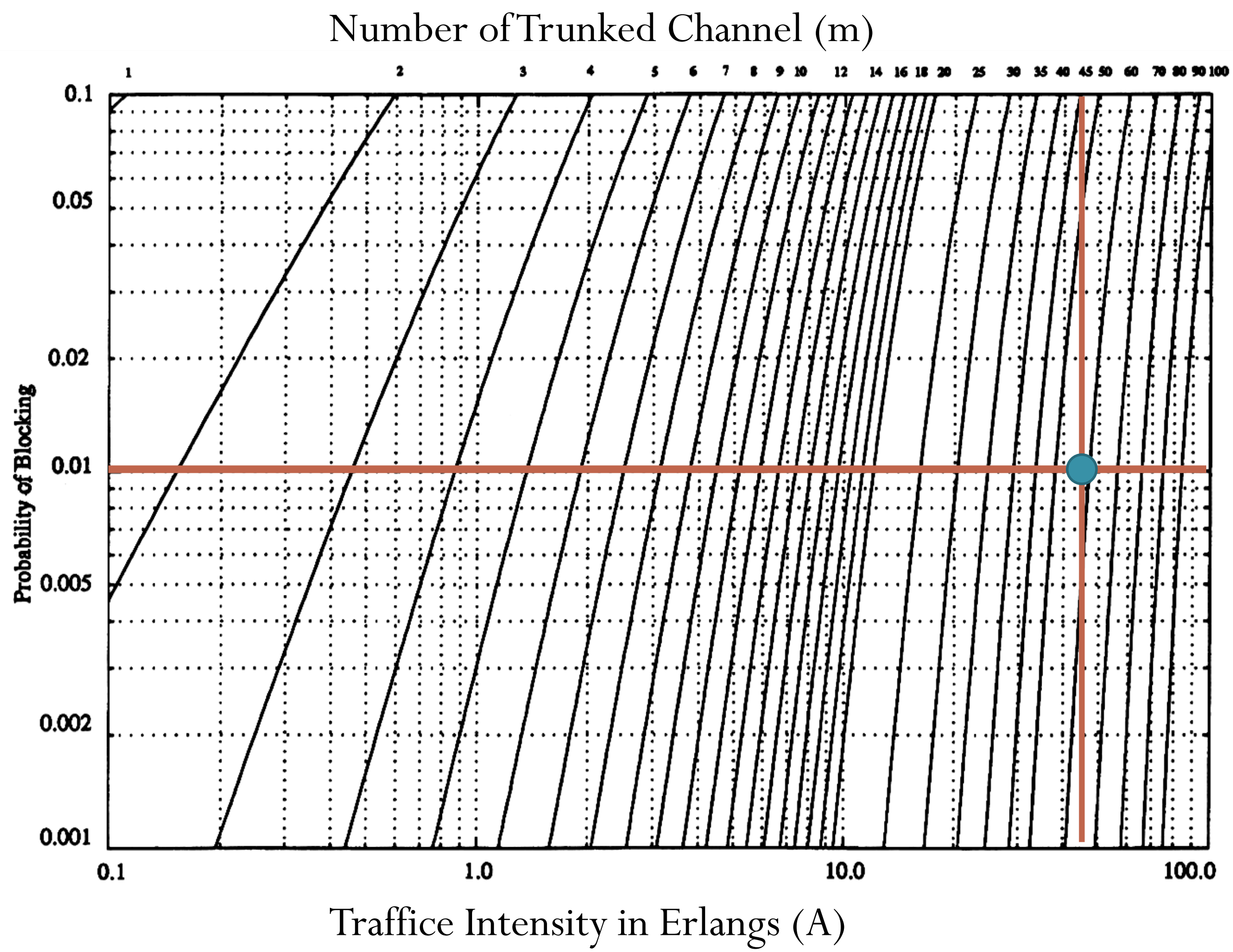
$$A \approx 4 \Rightarrow n \approx 40 \text{ users}$$

Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes
  - the probability of blocking is to be no more than 1%.
- If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell.
- From the Erlang B formula, can handle 44.2 Erlangs or **1326 calls per hour**.

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.1: Erlang B

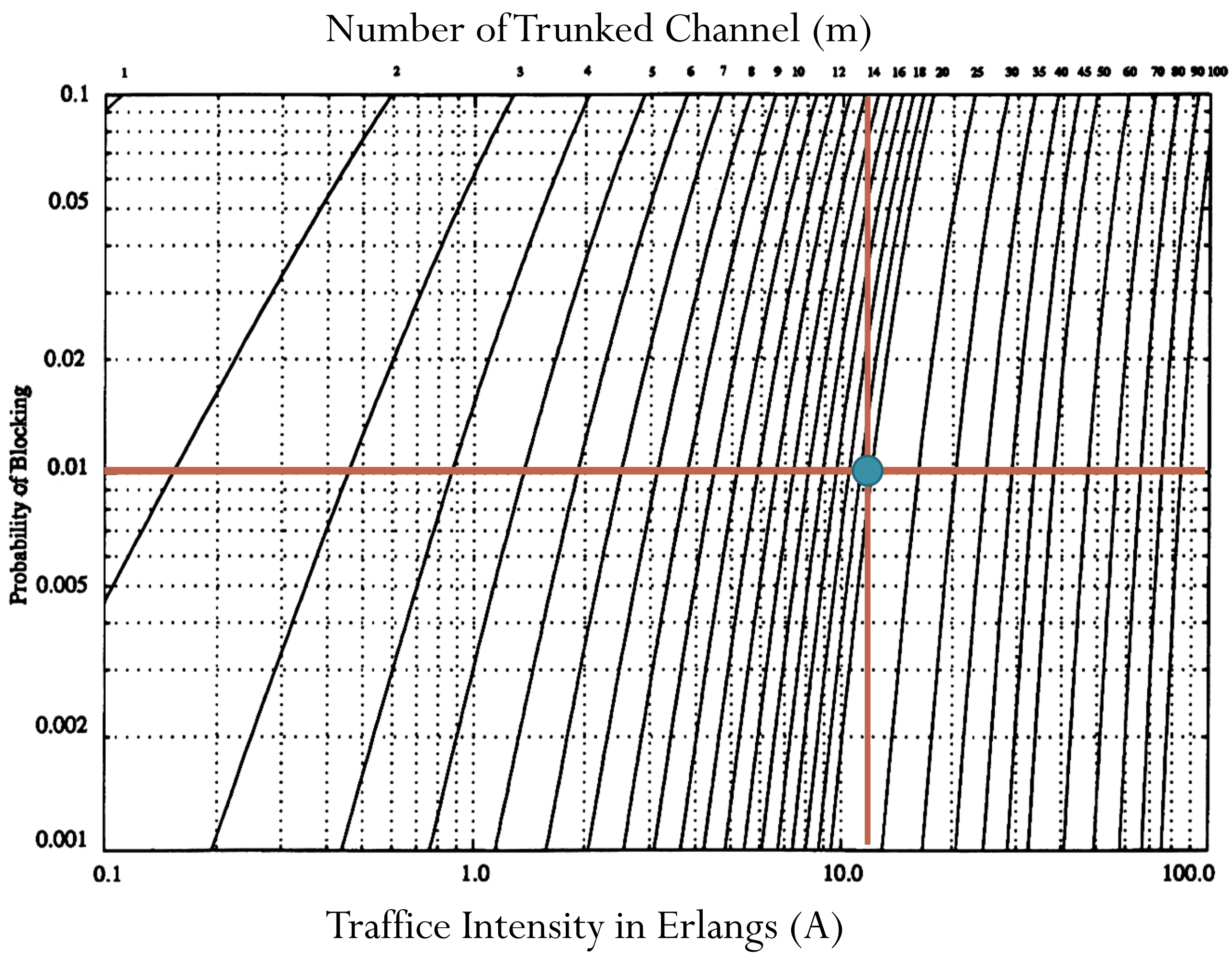


Example 2.2

- Now employing **120° sectoring**, there are only 19 channels per sector (57/3 antennas).
- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 =$ **1008 calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the SIR for each user in the system.

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.2: Erlang B



Erlang B Trunking Efficiency

Table 3.4 Capacity of an Erlang B System

Number of Channels C m	Capacity (Erlangs) for GOS			
	1% = 0.01	= 0.005	= 0.002	0.1% = 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

[Rappaport, 2002, Table 3.4]

Summary of Chapter 2: Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

"Capacity" $C = \dfrac{A_{total}}{A_{cell}} \times \dfrac{S}{N}$

Tradeoff

Path loss exponent

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

Omni-directional: K = 6

120° Sectoring: K = 2

60° Sectoring: K = 1

Trunking

m = # trunked channels

Call blocking probability

$$P_b = \frac{\dfrac{A^m}{m!}}{\displaystyle\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Erlang-B formula

λ = Average # call attempts/requests per unit time

A = traffic intensity or load [Erlangs] $= \dfrac{\lambda}{\mu}$

$\dfrac{1}{\mu} = H$ = Average call length

Example 3 (1)

- System Design
- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20 \times 10^6}{2 \times 25 \times 10^3} = 400 \text{ channels}$$

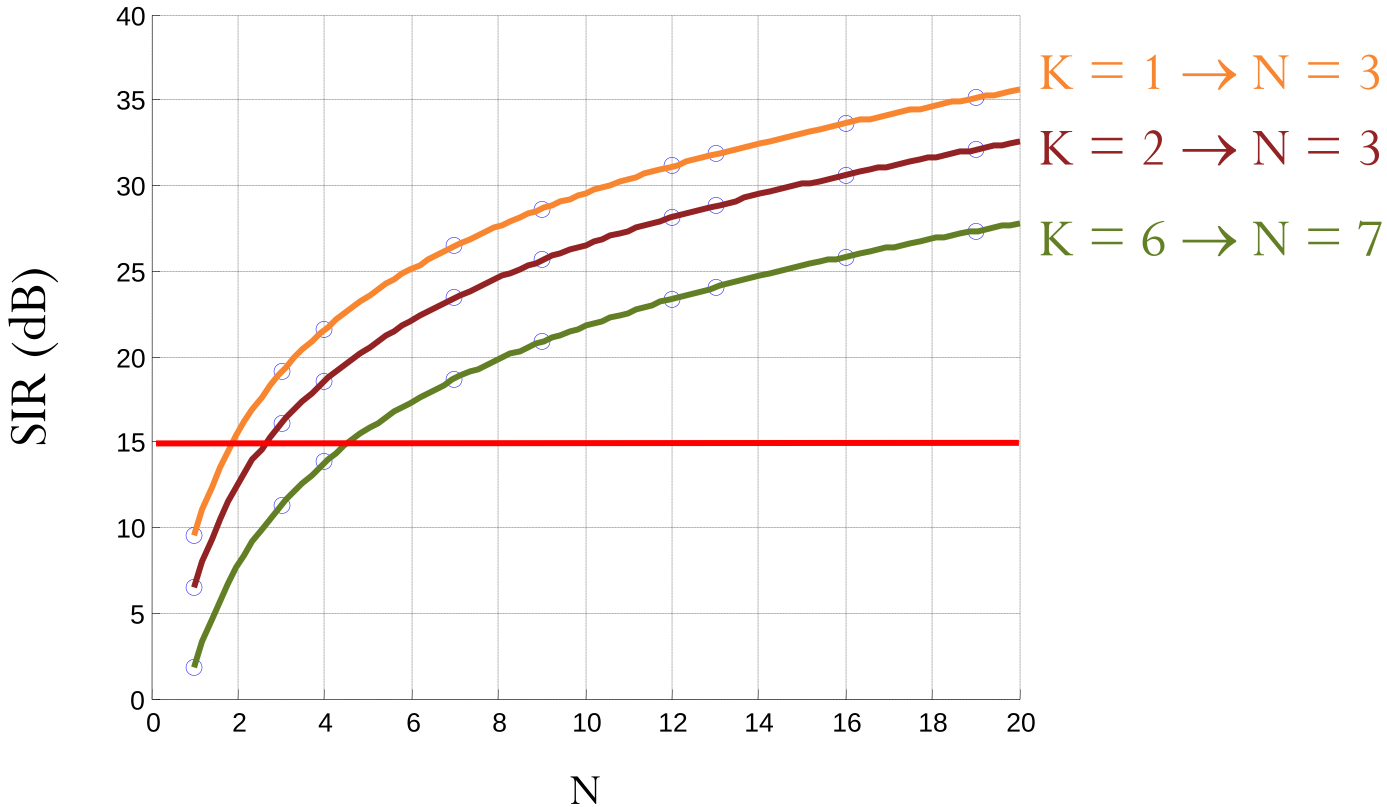
- $\gamma = 4$
- Design requirements:
  - SIR $\geq$ 15 dB
  - $P_b \leq 5\%$

Example 3 (2)

- SIR ≥ 15 dB

$$\frac{S}{I} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
    N = [1, 3, 4, 7, 9, 12, 13, 16, 19];
    SIR = 10*log10(1/K*((sqrt(3*N)).^y))
    plot(N,SIR,'o')
    N = linspace(1,20,100);
    SIR = 10*log10(1/K*((sqrt(3*N)).^y));
    plot(N,SIR)
end
```



Example 3 (3)

	Omnidirectional	Sectoring (120°)	Sectoring (60°)
K	6	2	1
N	7	3	3
SIR [dB]	18.7	16.1	19.1
#channels/cell	$400/7 = 57$	$400/3 = 133$	$400/3 = 133$
#sectors	1	3	6
#channels/sector	57	$133/3 = 44$	$133/6 = 22$
A [Erlangs]/sector	51.55	38.56	17.13
A [Erlangs]/cell	51.55	$38.56 \times 3 = 115.68$	$17.13 \times 6 = 102.78$
#users/cell	18558	41645	37001

Assume that each user makes 2 calls/day and 2 min/call on average $\rightarrow$ 1/360 Erlangs.

Make sure that you understand where numbers in this table come from!